Why is $Var(x) = \frac{(b-a)^2}{12}$ for a uniformly distributed variable?

$$VAR(x) = \int_a^b (x-\mu)^2 f(x)\,dx$$

$$= \int_a^b \left(x - \frac{a+b}{2}\right)^2 \cdot \frac{1}{b-a}\,dx$$

$$= \int_a^b \left[x^2 - 2x\frac{a+b}{2} + \left(\frac{a+b}{2}\right)^2\right]\frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\int_a^b \left[x^2 - x(a+b) + \left(\frac{a+b}{2}\right)^2\right]dx$$

$$= \frac{1}{b-a}\left(\frac{x^3}{3} - \frac{x^2}{2}(a+b) + \left(\frac{a+b}{2}\right)^2 x\right)\Bigg|_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^3}{3} - \frac{b^2}{2}(a+b) + \left(\frac{a+b}{2}\right)^2 b\right] - \frac{1}{b-a}\left[\frac{a^3}{3} - \frac{a^2}{2}(a+b) + \frac{(a+b)^2}{2}a\right]$$

$$= \frac{1}{b-a}\left[\frac{b^3}{3} - \frac{a^3}{3} - \frac{b^2(a+b)}{2} + \frac{a^2(a+b)}{2} + \left(\frac{a+b}{2}\right)^2 b - \left(\frac{a+b}{2}\right)^2 a\right]$$

$$= \frac{1}{b-a}\left[\frac{b^3-a^3}{3} + \frac{(a^2-b^2)(a+b)}{2} + (b-a)\left(\frac{a+b}{2}\right)^2\right]$$

$$= \frac{1}{b-a}\left[\frac{4b^3-4a^3}{12} + \frac{6(a^2-b^2)(a+b)}{12} + \frac{3(b-a)(a+b)^2}{12}\right]$$

$$= \frac{1}{b-a}\left[\frac{b^3-a^3+3a^2b-3b^2a}{12}\right] = \frac{1}{b-a}\left[\frac{(b-a)^3}{12}\right]$$

$$= \frac{(b-a)^2}{12}$$