BurkeyAcademy: Derivation of OLS

OLS DERIVE.PDF

Simple OLS Formulas for B_0 and B_1

$$\text{Min} \sum (e_i)^2 \qquad \text{Min} \sum (Y_i - \hat{Y}_i)^2 \qquad \hat{Y}_i := \hat{\beta}_0 + \hat{\beta}_1 X_i$$

$$* \text{Min} \sum (Y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$* \frac{\partial}{\partial \beta_0}: 2 \sum (Y_i - \beta_0 - \beta_1 x_i)(-1) = 0$$

$$\frac{\partial}{\partial \beta_1}: 2 \sum (Y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0$$

$$\sum Y_i - \sum \beta_0 - \sum \beta_1 x_i = 0$$

$$\sum \beta_0 = \sum Y_i - \sum \beta_1 x_i$$

$$\frac{n \beta_0}{n} = \frac{\sum Y_i}{n} - \frac{\sum \beta_1 x_i}{n} \Rightarrow \boxed{\beta_0 = \bar{Y} - \beta_1 \bar{X}}$$

$$\sum (Y_i - [\bar{Y} - \beta_1 \bar{X}] - \beta_1 x_i)\, x_i = 0$$

$$\sum (Y_i x_i - [\bar{Y} - \beta_1 \bar{X}] x_i - \beta_1 x_i x_i) = 0$$

$$\sum Y_i x_i - \sum \bar{Y} x_i + \sum \beta_1 \bar{X} x_i - \sum \beta_1 x_i x_i$$

$$* \sum (Y_i - \bar{Y}) x_i = \beta_1 \sum (x_i - \bar{X}) x_i = 0$$

$$* \sum (Y_i - \bar{Y})(x_i - \bar{X}) = \beta_1 \sum (x_i - \bar{x})(x_i - \bar{X})$$

$$\hat{\beta}_1 = \frac{\sum (Y_i - \bar{Y})(x_i - \bar{X})}{\sum (x_i - \bar{X})^2}$$

$$x\sum(Y_i-\bar{Y})(x_i-\bar{X}) = \sum Y_i x_i - \sum Y_i \bar{X} - \sum \bar{Y} x_i + \sum \bar{Y}\bar{X}$$