BurkeyAcademy

BY NC SA

Principal-Agent Problem: Principal Hires an Agent (Any Contract, in General)

The "Problem" can be **Hidden Action** or **Hidden Information**

Hidden Action: Moral Hazard
You might steal or be lazy

Hidden Information: Adverse Selection
You might not be qualified for the job

Consider a moral hazard problem. A principal hires an agent to mine some gold. The output ($\boldsymbol{Q}$) depends on **both** the agent's effort level (***c***: **Hi** (1) or **Lo** (0)) **and** Random Chance. The agent gets paid wages (***w***). The principal's utility function is

$$U_p\,(Q, w) = Q - w$$

The agent's utility function is

$$U_a\,(w, c) = \sqrt{w} - c$$

c is the cost of exerting effort. Agent has reservation utility of $\underline{U_a}$=10. Output q takes two values:

$$Q = \begin{cases} 400 \text{ with } p \\ 100 \text{ with } 1-p \end{cases}$$ Agent (High effort, c =1), $p=\frac{2}{3}$ Or (Low effort, c =0), $p=\frac{1}{3}$

Mechanism Design: Design the Best Contract to Induce the desired Effort Level and Maximize Profits
Three kinds of Equations:

1) **Profit Function**, e.g. $\max\limits_{w(\cdot)} E[Q(\text{effort, random}) - w(Q(\text{effort, random})]$
2) **Participation Constraint**: Agent will reject contract if it does not provide at least Reservation Utility Level
3) **Individual Rationality Constraint**: The Agent must choose the Effort Level we want

Several Solution Approaches: **1)** Optimal Control Theory 😱

2) Grossman and Hart (1983) Econometrica "An Analysis of The Principal-Agent Problem"
or Fudenberg and Tirole (1990) Econometrica "Moral Hazard and Renegotiation in Agency Contracts"
But the Final Publication version cuts this nice breakdown out!

See the Working Paper Version from **1988: MIT Economics Working Paper # 494

> ***"To solve the principal-agent model with commitment, Grossman and Hart use a three-step procedure. The first step is to 'characterize the set of incentive-compatible contracts that implement a given level of effort, or a given distribution over levels of effort. Next, find the element of this set that implements the desired distribution at the least cost to the principal. One expects that this least-cost contract will give the agent zero ex-ante rent, i.e. the agent's individual rationality constraint will bind. Finally, choose the distribution over effort that maximizes the difference between the principal's expected revenue and the cost of the agent's compensation."***

Effort is Observable!

$U_p\ (Q,w) = Q - w$ $\qquad\qquad$ $U_a\ (w,c) = \sqrt{w} - c$

c is the cost of exerting effort. $\underline{U_a}$=10 (reservation utility)

$Q = \begin{cases} 400 \text{ with } p \\ 100 \text{ with } 1-p \end{cases}$ Agent (High effort, c =1), $p=\frac{2}{3}$ Or (Low effort, c =0), $p=\frac{1}{3}$

Expected Value: $\boldsymbol{E(x) = \sum p_i\, x_i}$ $\qquad$ Expected Utility: $\boldsymbol{E(U) = \sum p_i\, U_i}$

A) Derive the 1st best contract (all actions observable, so principal knows the effort level)

How can we get Low Effort?

How can we Get Hi Effort?

So, which one is Best for the Principal?

$U_p\ (Q,w) = Q - w$ <u>Find Expected Value</u>

Effort is NOT Observable!

$U_p\,(Q, w) = Q - w$ $\qquad\qquad$ $U_a\,(w, c) = \sqrt{w} - c$

c is the cost of exerting effort. $\underline{U_a}$=10 (reservation utility)

$Q = \begin{cases} 400 \text{ with } p \\ 100 \text{ with } 1-p \end{cases}$ Agent (High effort, c =1), $p=\frac{2}{3}$ Or (Low effort, c =0), $p=\frac{1}{3}$

Expected Value: $\boldsymbol{E(x) = \sum p_i\, x_i}$ Expected Utility: $\boldsymbol{E(U) = \sum p_i\, U_i}$

B) Now, let's look at the 2nd best contract where only the output can be observed, but not the effort level. The wage can only be dependent on the OUTPUT.

<u>How can we get Low Effort?</u>

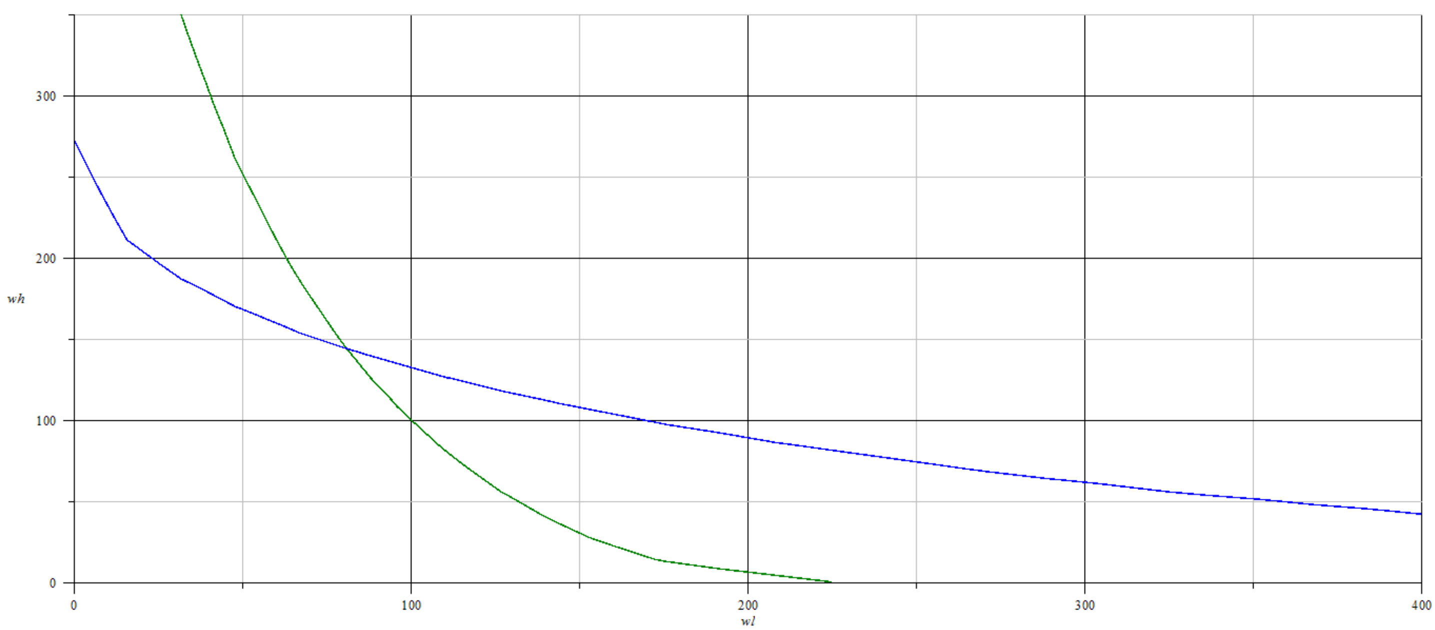
<u>How can we Get Hi Effort?</u>

<u>Cheapest way to get Lo Effort?</u>

Expected cost of wages: $\frac{2}{3}w_L + \frac{1}{3}w_H$

<u>Cheapest way to get Hi Effort?</u>

Expected cost of wages: $\frac{1}{3}w_L + \frac{2}{3}w_H$



Solve $\frac{1}{4}w_L - 16.5w_L^{.5} + 1089/4 = 4w_L - 120w_L^{.5} + 900$

or, $3.75w_L - 103.5w_L^{.5} + 627.75 = 0$

So, which one is Best for the Principal?

$U_p\,(Q, w) = Q - w$