Rough Draft

The Mother of All Elasticity Handouts! See videos at www.burkeyacademy.com © Mark L. Burkey

Introductory elasticity playlist: https://www.youtube.com/playlist?list=PL64670196A7BD80AB Advanced elasticity video: TBA

Introductory Elasticity Bullet Points:

- Elasticity measures how **responsive** ***B*** is to a change in ***A***
  - E.g., how responsive ***quantity*** is to ***price***
  - Think of ***A*** as a cause, ***B*** as the effect
- It is always measured as a ratio, with ***B*** in the numerator
- It is always a ***ratio*** of % changes
- An elasticity is **unit free**: i.e., it is not measured in %, $, etc.
- They are always conveniently named:

  A elasticity of ***B*** = $\frac{\%\Delta B}{\%\Delta A}$ (first word in the denominator)

  Example: ***Price*** goes up by 10% and ***Quantity*** goes down by 5%

  Price elasticity of ***Demand*** = $\frac{-5\%}{10\%} = -0.5$

  (the % signs cancel, -0.5 is unit free) "The elasticity is -0.5."
- If the magnitude (*absolute value*) of the ***numerator (effect)*** is larger than the magnitude of the ***denominator (cause)***, the |Elasticity|>1, and we say relationship is "elastic" (responsive)
- |Elasticity|<1 "inelastic", |Elasticity|=1 "unit elastic", |E|=0 "perfectly inelastic", |E|=∞ "perfectly elastic"
- **Interpreting an elasticity:** If the elasticity = -0.5, notice that

  -0.5= $\frac{\%\Delta B}{\%\Delta A}$. Assume %Δ***A*** =1% for simplicity. Then: $\frac{\%\Delta B}{1\%} = \frac{-0.5\%}{1\%}$.

  "For each **+1%** change in ***A*** there will be a **-0.5%** change in ***B***."

  "For a **-10%** Δ in ***A*** there will be a **+5%** Δ in ***B***." (multiply each by -10)

  "For +2% Δ in ***B*** there is a (solve **-0.5**= $\frac{+2\%}{\%\Delta A}$) %Δ*A*." *%Δ***A*=4%**

Midpoint Formula for $\%\Delta x = \frac{\Delta x}{average\ x} * 100$

Example: Price goes from $100 to $80. $\%\Delta P = \frac{\Delta P}{average\ P} * 100$

$= \frac{80-100}{\left(\frac{80+100}{2}\right)} * 100 = \frac{-20}{90} * 100 = -22.2\%$. Midpoint method is "better" because it gives the **same magnitude** for %Δ if we go from 80 to 100 or 100 to 80. The "normal" method gives +25% in the first case, -20% in the latter.

Most Common Elasticity in Economics: **Price Elasticity of Demand (ε_d)**

- Are people very responsive to price changes, or not?
- Always negative or zero, since **Giffen Goods do not exist**
- Since the sign is **always** negative, often economists are lazy and will say "The price elasticity of demand is 2.", and expect you to know they mean "-2".
- "Total Revenue Test": If you raise price and total revenue increases, %ΔQ<%ΔP, so inelastic.
- For **linear** demand: Price E of Demand = -1 at midpoint (Choke price/2). Elastic at higher prices, elastic below. Total Revenue maximized at midpoint.
- Goods are **more** elastic (people can respond more to price changes) over **longer** time frames, if there are **lots of** substitutes, if the good is a **large** portion of total budget, if it is **necessary** for life.

Other Common Elasticities:

- Income elasticity of demand: $\frac{\%\Delta Q}{\%\Delta I}$ **(ε_I)**
  - + normal goods, - inferior
- Cross-Price elasticity of demand: $\frac{\%\Delta Q_x}{\%\Delta P_y}$ **(ε_{xy} or ε_{yx})** notation a bit iffy
  - +substitutes, - complements
- Price elasticity of Supply: $\frac{\%\Delta Q_s}{\%\Delta P}$ **(ε_s)**
- But, it is not limited to these. ANY cause/effect relationship can be estimated as an elasticity.

E.g.: Travel cost elasticity of demand for liquor (Burkey 2010)

$\frac{\%\Delta Quantity_{liquor\ bought}}{\%\Delta Access\ Cost\ to\ Liquor\ Store}$ =-0.12 ≠ $\frac{\%\Delta Quantity_{liquor\ bought}}{\%\Delta Price\ of\ Liquor}$ ≈-1.5

This elasticity measures how responsive purchases of liquor (e.g. rum, whiskey) are to the distance people must travel to purchase it. Not the same thing as the price elasticity of demand for liquor! (Much, much smaller!)

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Advanced Elasticity Stuff:

Elasticity at a point: If we know the slope of the relationship between A and B, we can calculate the elasticity at a point. Derivation:

Since $\%\Delta x = \frac{\Delta x}{average\ x}*100$ and Elasticity = $\frac{\%\Delta B}{\%\Delta A}$, replace "average *x*" with *x*.

$\frac{\%\Delta B}{\%\Delta A} = \frac{\frac{\Delta B}{B}*100}{\frac{\Delta A}{A}*100} = \frac{\Delta B}{B}\frac{A}{\Delta A} = \frac{\Delta B}{\Delta A}\frac{A}{B}$. E.g., **Price elasticity of demand** = $\frac{\Delta Q_d}{\Delta P}\frac{P}{Q_d}$

Using Calculus: Price elasticity of demand $\varepsilon_d = \frac{\partial Q_d}{\partial P}\frac{P}{Q_d}$

So, if you have any demand function, take the derivative and multiply by P/Q to get the price elasticity of demand.

Examples:

***Linear Demand: Q=100-2P Price E Elasticity** = $\frac{\partial Q_d}{\partial P}\frac{P}{Q_d}$= $-2\frac{P}{Q} = \frac{-2P}{101-2P}$

***Cobb-Douglas Demand Function:** $Q_x=3P_x^{-1.2}P_y^{.5}I^{1.3}$

$\varepsilon_d = \frac{\partial Q_x}{\partial P_x}\frac{P_x}{Q_x} = (-1.2)(3)P_x^{-2.2}P_y^{.5}I^{1.3}\frac{P_x}{Q_x}$, **substitute in equation for Q_x,**

→ $\frac{(-1.2)(3)P_x^{-2.2}P_y^{.5}I^{1.3}P_x}{(3)P_x^{-1.2}P_y^{.5}I^{1.3}}$ → Everything except the -1.2 cancels.

Lesson: In this kind of demand function, the exponents on P_x, P_y, and I are the own-price, cross-price, and Income elasticities of demand!

Try it yourself! $\varepsilon_I = \frac{\partial Q_x}{\partial I}\frac{I}{Q_x}$ **=1.3** $\varepsilon_{xy} = \frac{\partial Q_x}{\partial P_y}\frac{P_y}{Q_x}$ **= 0.5**

Simplest version of this kind of demand function is $Q=P^{-1}$, ε_d always -1.

Elasticity and Econometrics:

The Cobb-Douglas demand function is convenient for estimating demand functions, but it assumes that elasticity never changes. While it is superior to estimating linear demand functions (because linear demand functions are inconsistent with any sensible utility function), it is not necessarily correct. However, it is very commonly used because it easily gives elasticity estimates. Take the logarithm of both sides: **$\ln(Q_x)=\ln(3)-1.2\ln(P_x)+.5\ln(P_y)+1.3\ln(I)$.** Before estimating, we write:

$\ln(Q_x)=\ln(B_0)+B_1\ln(P_x)+B_2\ln(P_y)+B_3\ln(I)$

***In short, coefficients from a log-log regression give you elasticities!**

Elasticity with analytical functions:

Suppose we had a Marshallian Demand Function from a Cobb-Douglas Utility (not Cobb-Doulas Demand!) maximization problem

Max $U = x^{\alpha}y^{1-\alpha}$ s.t. px+qy=M

$x^*=\frac{\alpha M}{p}$ So, $\varepsilon_d = \frac{\partial x}{\partial p}\frac{p}{x} = -1 * \frac{\alpha M}{p^2}\left(\frac{p}{x}\right)$, **substitute in x* for x**

→ $=\frac{-\alpha M}{p^2}\left(\frac{p}{\frac{\alpha M}{p}}\right)$, everything cancels! **$\varepsilon_d$=-1!**

Try it yourself! $\varepsilon_I = \frac{\partial x}{\partial M}\frac{M}{x}$ **=1.0** $\varepsilon_{xy} = \frac{\partial x}{\partial q}\frac{q}{x}$ **= 0**

What if utility involved perfect complements?

$$u = Min\ [\alpha x, (1-\alpha)y] \qquad x^* = \frac{M}{p+\frac{\alpha}{1-\alpha}q}$$

$\varepsilon_d = \frac{\partial x}{\partial p}\frac{p}{x} = \frac{-p}{p+\frac{\alpha}{1-\alpha}q}$ ε_I**=1** $\varepsilon_{xy} = \frac{-\alpha q}{(1-\alpha)(p+\frac{\alpha q}{1-\alpha})}$

Video for Derivarions of the Cobb Douglas and Perfect Complements Elasticities: https://youtu.be/ATSM8vjMY7k