The Mother of All Consumer Theory Handouts! See videos at www.burkeyacademy.com © Mark L. Burkey

Marshallian Demand

Max U(x,y) s.t. $P_x x + P_y y = \bar{B}$ (or $\bar{I}$, $\bar{m}$)

Example: Max $U = x^{.3}y^{.7}$ s.t. $P_x x + P_y y = \bar{B}$ **e.g., $\bar{B}$ = \$100**

Solve using calculus, or Solve $MRS_{xy} = P_x/P_y$, $P_x X + P_y Y = \bar{B}$

$MRS_{xy} = MU_x/MU_y$, $MU_x = \frac{\partial U}{\partial x}$

Marshallian Demands for x & y **How much x and y should I buy seeing these prices with this budget to max U?**

$x^*(P_x, P_y, \bar{B})$, $y^*(P_x, P_y, \bar{B})$

Example: $x^* = .3\bar{B}/P_x$ $y^* = .7\bar{B}/P_y$

Substitute x and y* into the utility function*

Roy's Identity $\frac{-\partial U^*}{\partial P_x} / \frac{\partial U^*}{\partial B} = x^*$ ***similar for y*, use P_y***

Indirect Utility Function: **Direct utility converts x and y into happiness. Indirect uses prices and budget to do this, *indirectly*.**

Utility* = $V(P_x, P_y, \bar{B})$ Example: $U^* = .5423\left(\frac{B}{P_x}\right)^{.3}\left(\frac{B}{P_y}\right)^{.7}$

Hicksian (Compensated) Demand

Min $B \equiv (P_x x + P_y y)$ s.t. $U(x,y) = \bar{U}$ (or $=U_o$)

Example: Min $P_x x + P_y y$ s.t. $x^{.3}y^{.7} = \bar{U}$

Solve using calculus or solve *$MRS_{xy} = P_x/P_y$, $U(x,y) = \bar{U}$* ***For example, $x^{.3}y^{.7} = 10$***

Hicksian (Compensated) Demand **How much x and y should I buy to get $\bar{U}$ the cheapest way?**

$x^c(P_x, P_y, \bar{U})$, $y^c(P_x, P_y, \bar{U})$ (or $x^* = H(P_x, P_y, \bar{U})$)

Example: $x^c = \bar{U}\left(\frac{3P_y}{7P_x}\right)^{.7}$, $y^c = \bar{U}\left(\frac{7P_x}{3P_y}\right)^{.3}$

Substitute x^c and y^c into the Budget equation

Shephard's Lemma $\frac{\partial E}{\partial P_x} = x^c$ ***similar for y^c, use P_y***

Expenditure Function **How much does it cost to get $\bar{U}$ in the cheapest way given P_x & P_y?**

$E(P_x, P_y, \bar{U}) = P_x x^c + P_y y^c$

Example: $P_x \bar{U}\left(\frac{3P_y}{7P_x}\right)^{.7} + P_y \bar{U}\left(\frac{7P_x}{3P_y}\right)^{.3} = 10\bar{U}\left(\frac{p_y}{7}\right)^{.7}\left(\frac{p_x}{3}\right)^{.3}$

Links between the two sides

Plug Expenditure in for B in Marshallians, get Hicksians!

Plug Indirect in for U in Hicksians, get Marshallians!

Solve Indirect for B (Invert)

Solve Expenditure for U (Invert)

<u>Indirect Money Metric Utility Function</u>: Take the Indirect Utility function and put bars on $\bar{P}_x, \bar{P}_y, \bar{B}$. Substitute this fn. into the expenditure function for $\bar{U}$. This will tell you how much \$ you need to get the same utility you **<u>were</u>** getting at $\bar{P}_x, \bar{P}_y, \bar{B}$ under new prices P_x & P_y. Example: IMM=$\frac{p_x^{.3}p_y^{.7}\bar{B}}{\bar{p}_x^{.3}\bar{p}_y^{.7}}$

<u>Money Metric Utility Function</u>: Plug the original utility function into the Expenditure function. This tells you "The mimimum \$ needed to get the same utility as I get from x_o and y_o. Example: MM=$1.842 y_o^{.7} x_o^{.3} p_y^{.7} p_x^{.3}$

<u>Legend</u>: Green indicates various common symbols others might use for the same thing.

B, I, m: ***B**udget, **I**ncome, **m**oney:* If they are a free variable, or the **outcome**.

$\bar{B}$ or $(\bar{I}, \bar{m}, or\ B_o)$: The bar or subscript, e.g. B_o (read "B nought") means **fixed** level of income- the main constraint. Should think of as usually fixed.

U: Utility, happiness, satisfaction. $\bar{U}$ or U_o : A fixed level of Utility that we want to achieve. U*: The optimal level of utility possible

x and y: Two goods we are consuming to produce Utility

x* and y* : Optimal levels of x and y that maximize utility; demand for x and y.

x^c , y^c : Hicksian demands for x and y; optimal levels of x and y that attain $\bar{U}$ the cheapest way

P_x and P_y : Prices of the goods x and y. Overview Video: https://www.youtube.com/watch?v=T-q_ZXmfiLI